Keywords
analytic functions, elliptical holes, stress intensity factor, conformal mapping functions, stress components.

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Stress-Strain State Of a Polygon With Cracks Caused By Thermal Stresses

Abstract
When addressing engineering design challenges, engineersfrequently encounter various structural defects—includingcracks—that can lead to the failure of machinery and equipment.This study examines a regular polygon affected by linear cracks caused by thermal stresses.
Since conformal mapping functions for such complex domains containing linear cracks were previously unknown, this problem had not been addressed in earlier studies. In the present paper, aplane elasticity problem involving a complex geometric configuration—specifically, a domain weakened by twoopenings with rectilinear cuts—is solved using the theory offunctions of a complex variable and conformal mapping functions.
The problem was addressed by solving a system of linear algebraic equations utilizing the theory of complex variables and Kolosov - Muskhelishvili potentials 7,8; the system was solvedby expanding the functions φ(z) and ψ(z) into series. Ananalytical solution was obtained, and subsequent studies will present numerical examples illustrating key aspects of the theory. Solving these linear algebraic equations yielded thecoefficients of the analytic functions; subsequently, establishedformulas from the theory of elasticity were used to determine the stress components at characteristic points of the cross-sections.This problem is being solved for the first time, as no conformalmapping functions had previously been found for such shapes—specifically, for the complex geometric configuration of thebody in question.

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1. INTRODUCTION

1.1 Aim and objectives of the study.

Stress concentrations arise in machine components and mechanisms of complex design that are weakened by various types of openings and contain cracks. Our objective is to investigate the stress-strain state of an infinite strip containing two openings and two symmetric cracks. To achieve this, we have formulated two tasks. In the first task, we constructed a mapping function. This function was used to map complex configurations onto the unit circle.

In the second task, utilizing the conformal mapping function and regular analytic functions within the boundary conditions, we determined the stresses at characteristic points.

2. MATERIALS AND METHODS

The problem was solved by solving a system of linear algebraic equations using the theory of complex variables and potentials of Kolosov-Muskhelishvili. This paper presents the basic formulas for calculating stresses at the characteristic points of the cross-sections of the plate [6]. The system of linear algebraic equations was solved by expanding the functions φ (z), ψ (z) into a series [6, 7]. In the course of the study, an analytical solution was obtained, and numerical examples were given to illustrate important aspects of the theory. By solving these linear algebraic equations, the coefficients of the analytical function were found and the well-known formulas of the theory of elasticity were used to determine the stress components and the stress components were found at the characteristic points of the sections

2.1 Method for solving the problem

In the second problem, using analytic regular functions of the theory of the function of a complex variable, the presented problem is solved. Then we map these outlines to single circles. After complex mathematical calculations, we present this problem for two linear algebraic equations. And then we keep the first few terms (coefficients) of linear algebraic equations. After that, to determine the normal and tangential stresses, we accept special formulas from the plane problem of the theory of elasticity and determine at certain points the stress components

3. METHOD FOR SOLVING THE PROBLEM

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Consider a plate with a region S bounded externally and internally by contours l1 and l2, respectively. The inner region is an ellipse containing linear cracks along the real axis.

It is necessary to determine the distribution of thermal stresses within the region. To do this, the temperatures at the outer and inner boundaries must be known. A point heat source of power Q acts at the initial point.

It is known [2] that the temperature field of the plate in question is determined by integrating the heat conduction equation.

$$\Delta t = \frac{Q}{\lambda} f_1(x) f_2(y) \qquad (3.1)$$

Here, the Laplace operator is presented in the following form:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$$

Here, λ-is the thermal conductivity coefficient, and $f_1(y) = \delta(y - c)$ and $f_1(x) = \delta(x - d)$ - are Dirac delta functions, where c and d are the coordinates of the point heat source.

The general solution of the given equation (1) can be expressed in terms of analytic functions f(z) and f(z) of the complex variable $z = x + iy$ z as follows:

$$t = 2Ref(z) = f(z) + \overline{f(z)} \qquad (3.2)$$

This function f(z) is represented as follows:

$$f(z) = Aln(z - z_0) + Bln(z) + f_*(z) \qquad (3.3)$$

Both functions $f_*(z)$ are regular in the given region of the plate. Let us represent it as follows:

$$f_*(z) = \sum_{k=1}^{\infty} d_k^{(1)} \xi^{-k} + \sum_{k=1}^{\infty} b_k^{(1)} \left(\frac{z}{A_2}\right)^k \qquad (3.4)$$

Here

$$b_k^{(1)} = \sum_{n=k}^{\infty \; **} \beta_n^{(1)} a_{(n+k)/N}^{(1)}$$

The variable ξ is related to z by the dependence $\xi = F(z)$. Here, the exterior of the plate's outer contour is mapped onto the exterior of the unit circle. [1]

$$z = A_2\left(\xi + \frac{m_2}{\xi^{N-1}}\right) = A_2\xi \sum_{n=0}^{\infty} m_2^n \xi^{-nN} \qquad (3.5)$$

The shape of the contour is mapped onto the shape of the unit circle. [2]

$$z = A_1\xi \sum_{n=0}^{\infty} \Pi_n \xi^n = A_1\xi \sum_{n=0}^{\infty} \xi^n \sum_{k=0}^{\infty} \xi_{k-1} P_{n-k} \qquad (3.6)$$

$$P_n = \sum_{n_1=0}^{\infty \; *} m_1^{\frac{n-n_1}{2}} \gamma_{-1}^{n_1-n} l_n^{n-n_1}$$

All quantities $\Pi_n, \gamma_n, l_n^{(k)}$, etc., are defined according to [2].

$$f(t_1) = Aln(t_1 + z_0) + Blnt_1 + \sum_{k=1}^{\infty} \alpha_k^{(1)} \tau^{-k} +$$

$$+\sum_{k=1}^{\infty} H_1^{(1)}(k)\tau^k + \sum_{k=1}^{\infty} H_2^{(1)}(k)\tau^{-k} + \beta_0^{(1)} \quad na \quad l_1 \qquad (3.7)$$

$$f(t_2) = Aln(t_2 + z_0) + Blnt_2 + \sum_{k=1}^{\infty} N_k^{(1)}(k)\tau^{-k} +$$

$$+\sum_{k=1}^{\infty} N_2^{(1)}(k)\tau^k + \sum_{k=1}^{\infty} N_3^{(1)}(k)\tau^{-k} + \beta_0^{(1)} \quad na \quad l_2 \qquad (3.8)$$

Substituting (3.7) and (3.8) into (3.2) and equating the coefficients of like powers of τ, we obtain the following systems of linear algebraic equations for determining the unknown coefficients $B, \alpha_k^{(1)}, \beta_k^{(1)}$ and $\beta_0^{(1)}$.

$$2Alnz_0 + 2AV_1(0) + 2BlnA_1\Pi_0 + 2BV_3(0) + 2\beta_0^{(1)} = t_1 \qquad (3.9)$$

$$2AlnA_2 + 2Bln + 2BlnA_1\varPi_0 + 2BV_3(0) + 2\beta_0^{(1)} = t_1 \qquad (3.10)$$

$$H_1^{(1)}(k) + BV_3(k) + \alpha_k^{(1)} + H_2^{(1)}(k) = -A[V_1(k) + V_2(k)] \quad (3.11)$$

$$B\varepsilon_1(-1)^{k-1}\frac{m_2}{k} + N_1^{(1)} + N_2^{(1)}(k) + N_3^{(1)}(k) =$$

$$= -A\left[(-1)^{k-1}\frac{m_k}{k}\varepsilon_1 + V_4(k)\right] \qquad (3.12)$$

Here $$\varepsilon_1 = \begin{cases} 0 \ \ at \ \ k \neq N, \ \ 2N, \ldots., nN \\ 1 \ \ at \ \ k = N, \ \ 2N, \ldots., nN \end{cases}$$

The expressions for the unknown coefficients appearing in (3.9), (3.10), (3.11), and (3.12) are not presented here due to their cumbersome nature.

Knowing the temperature distribution in the plate under consideration, we determine the stress field (arising from the non-uniformity of this distribution) using formulas.[1]

$$\sigma_x^{(1)} + \sigma_y^{(1)} = -8G\delta_1\left[f(z) + \overline{f(z)}\right] \qquad (3.13)$$

$$-\sigma_x^{(1)} + \sigma_y^{(1)} + 2i\tau_{xy}^{(1)} = -8G\delta_1[\bar{z}f'(z) + F'(z)]$$

$$u^{(1)} + iv^{(1)} = 2\delta_1\left[\int\!\!\int f(z)\,dz + z\overline{f(z)} + F(\bar{z})\right] \qquad (3.14)$$

Here $\delta_1 = \alpha_t(1+\nu)/4$

f_z —функция, определяемая из условия однозначности перемещений.

It is known that, upon a single traversal of any closed loop l, only the quantities containing lnz acquire an increment.

Thus

$$\left[u^{(1)} + iv^{(1)}\right]_l = \left[\int\!\!\int f(z)dz + F(\bar{z})\right]_l = 0 \qquad (3.15)$$

Из этого условия определяется функции *f(z)*. Для рассматриваемой задачи функция *f(z)* имеет вид.

$$F(z) = Az_0 ln(\bar{z} - \bar{z}_0) - \alpha_1 A_1 ln\bar{z} \qquad (3.16)$$

To determine the second stress field, it is necessary to solve a plane problem—specifically, for the case where loads $(-R_{xn}^{(1)})$ and $(-R_{yn}^{(1)})$ are applied to the boundaries of the l_j region, which are otherwise free of external forces.

In this case, the problem reduces to determining the functions φ(z) and Ψ(z) from the boundary conditions.[3,7]

$$\varphi(t) + t\overline{\varphi'(t)} + \overline{\Psi(t)} = -i\int_{t_0}^{t}\left[R_{xn}^{(1)} + R_{yn}^{(1)}\right]ds \qquad (3.17)$$

t – affix of contour points

$$R_{xn}^{(1)} = \sigma_x^{(1)}cos(n,x) + \tau_{xy}^{(1)}cos(n,y)$$

$$R_{yn}^{(1)} = \sigma_y^{(1)} cos(n,y) + \tau_{xy}^{(1)} cos(n,x) \tag{3.18}$$

If a boundary of the plane region is rigidly clamped, then in the actual problem there should be no displacements along this boundary; however, based on (3.14), the displacements of the first field along this boundary are equal to φ(z) and Ψ(z). For the displacements in the actual body to be single-valued, the complex potentials on this clamped boundary must satisfy the following boundary conditions.

$$æ\varphi(t) - t\overline{\varphi'(t)} - \overline{\Psi(t)} = -2G\left(u^{(1)} + iv^{(1)}\right) \tag{3.19}$$

$$\varphi(z) = \sum_{k=1}^{\infty} \alpha_k^{(1)} \xi^{-k} + \sum_{k=1}^{\infty} b_k^{(1)} \left(\frac{z}{A_2}\right)^k + \beta_0^{(2)}$$

$$\Psi(z) = \sum_{k=1}^{\infty} c_k \xi^{-k} + \sum_{k=1}^{\infty} \lambda_k \left(\frac{z}{A_2}\right)^k + \beta_0^{(2)} \tag{3.20}$$

Using the mapping functions (3.5) and (3.6) on the contours, and after some transformations, we obtain the following expressions for φ(z).

$$\varphi(t) = \varphi(\tau) = \sum_{k=1}^{\infty} \alpha_k^{(1)} \tau^{-k} + \sum_{k=1}^{\infty} H_1^{(2)}(k)\, \tau^k + \beta_0^{(2)} +$$

$$+ \sum_{k=1}^{\infty} H_2^{(2)}(k)\, \tau^{-k} \qquad (l_1) \tag{3.21}$$

$$\varphi(t) = \varphi(\tau) = \sum_{k=1}^{\infty} N_1^{(2)}(k)\, \tau^{-k} + \sum_{k=1}^{\infty} N_2^{(2)}(k)\, \tau^k + \beta_0^{(2)} +$$

$$+ \sum_{k=1}^{\infty} N_3^{(2)}(k)\, \tau^{-k} \qquad (l_2) \tag{3.22}$$

Similarly, by substituting (3.5) and (3.6) into (3.20) for the inner and outer contours and performing some simplifications, we obtain the following expressions.

$$\overline{\Psi(t)} = \overline{\Psi(\tau)} = \sum_{k=0}^{\infty} c_k \tau^k + \sum_{k=0}^{\infty} E_9(k)\, \tau^{-k} + \sum_{k=0}^{\infty} E_{10}(k)\, \tau^k \qquad l_1 \tag{3.23}$$

$$\overline{\Psi(t)} = \sum_{k=1}^{\infty} N_7(k)\, \tau^k + \sum_{k=0}^{\infty} N_8(k)\, \tau^{-k} + \sum_{k=0}^{\infty} N_9(k)\, \tau^k \tag{3.24}$$

Taking into account the mapping functions (3.5) and (3.6), and after some mathematical manipulations, the expressions $t\overline{\varphi'(t)}$ appearing in equation (3.17) reduce to the following form on each of the contours l_j.

$$t\overline{\varphi'(t)} = \frac{\omega(\tau)}{\overline{\omega'(t)}}\overline{\varphi'(\tau)} = \tau^2\left[\sum_{k=0}^{\infty} E_2\,(k)\tau^{-k} + \sum_{k=0}^{\infty} E_3\,(k)\tau^{k}\right] +$$

$$+\tau^2\sum_{k=0}^{\infty} E_6\,(k)\tau^{-k} + \tau^3\left[\sum_{k=0}^{\infty} E_7\,(k)\tau^{-k} + \sum_{k=0}^{\infty} E_8\,(k)\tau^{k}\right] \qquad l_1 \quad (3.25)$$

$$\overline{\varphi'(t)} = \left(\tau + \frac{m_2}{\tau^{N+1}}\right)\left[-\sum_{k=2}^{\infty} N_4\,(k)\tau^{k} + \sum_{k=0}^{\infty} N_5\,(k)\tau^{-k}\right] +$$

$$+\left(\tau + \frac{m_2}{\tau^{N+1}}\right)\sum_{k=0}^{\infty} N_6\,(k)\tau^{k} \qquad l_2 \quad (3.26)$$

Substituting (3.21), (3.22), (3.23), (3.24), (3.25), and (3.26) into the boundary condition (3.17) and equating the coefficients of like powers, we obtain a system of linear algebraic equations.

By retaining the first few equations from each system of linear algebraic equations and solving them simultaneously, we determine the unknown expansion coefficients for functions φ(z) and Ψ(z). Once the complex potentials φ(z) and Ψ(z) have been determined, the components of the second stress field ($\sigma_x^{(2)}$, $\sigma_y^{(2)}$, and $\tau_{xy}^{(2)}$) and the displacements ($u^{(2)}$ and $v^{(2)}$) are found using formulas [1,2,3].

$$\sigma_x^{(2)} + \sigma_y^{(2)} = 4Re\varphi'(z) \qquad (3.27)$$

$$-\sigma_x^{(2)} + \sigma_y^{(2)} + 2i\tau_{xy}^{(2)} = 2[\bar{z}\varphi''(z) - \Psi'(z)] \qquad (3.28)$$

$$2G\left(u^{(2)} + iv^{(2)}\right) = æ\varphi(z) - z\overline{\varphi'(z)} - \overline{\Psi(z)} \qquad (3.29)$$

The sought-after actual stress and displacement fields in real bodies are equal to the sum of the first and second stress and displacement fields.
Next, the thermal stress intensity factors at the endpoints of the crack are determined using known formulas.
The primary objective of the study was to investigate the stress distribution around a crack within a temperature field and to analyze the temperature dependence of the stress intensity factor at the crack tip.

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